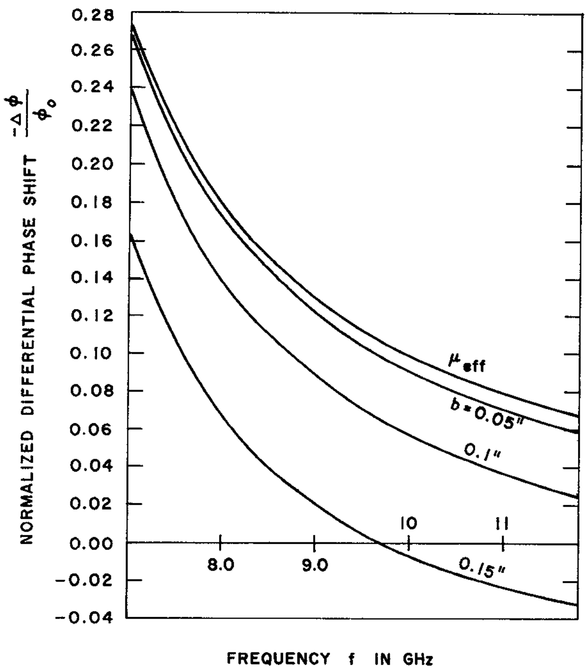

Fig. 2. Normalized differential phase shift as a function of frequency with plate spacing as the parameter.

cutoff, is to produce variable insertion phase which changes with the mutual series-impedance term. The insertion phase increases if the cutoff mode is TM-like and decreases if the cutoff mode is TE-like. This points out the fundamental difference between the Reggia–Spencer and the TEM phase shifters.

The TEM phase shifter was analyzed by applying the generalized telegraphists' equations in order to evaluate the effects of coupling to many higher order modes. A quadratic approximation was found which demonstrates the reduction in phase shift from the μ_{eff} limit, which is caused by coupling to higher order modes. When $kb \approx 1$ this reduction is negligible. The frequency dependence of the differential phase shift will always be greater than that given by the μ_{eff} limit. Furthermore, positive and negative phase shifts can occur in different frequency ranges for the TEM device depending upon plate spacing.

Appendix

The polynomial equation for the eigenvalues is given by (14) and is repeated here for convenience

$$\left(\mu - \frac{\beta^2}{k^2}\right) - a_{12}^2 \sum_{n=1,3,5,\cdots}^{\infty} \frac{r_n}{n^2(\mu r_n - \beta^2/k^2)} = 0. \tag{A1}$$

When the plate spacing approaches zero, the r_n approach infinity and (A1) reduces to

$$\left(\mu - \frac{\beta^2}{k^2}\right) - \frac{8}{\pi^2}\frac{\kappa^2}{\mu} \sum_{n=1,3,5,\cdots}^{\infty} \frac{1}{n^2} = 0. \tag{A2}$$

Solution of this yields

$$\frac{\beta^2}{k^2} = \frac{\mu^2 - \kappa^2}{\mu} = \mu_{\text{eff}} \tag{A3}$$

which is the expression derived by Suhl and Walker [1].

A better two-mode approximation than (15) in the text is obtained by rewriting (A1) as

$$\left(\mu - \frac{\beta^2}{k^2}\right) - a_{12}^2 \frac{r_1}{\mu r_1 - \beta^2/k^2} - a_{12}^2 \sum_{n=3,5,7,\cdots}^{\infty} \frac{r_n}{n^2(\mu r_n - \beta^2/k^2)} = 0. \tag{A4}$$

If only the dominant mode is of interest, $\beta/k < 1$ and the last term of (A4) may be written

$$a_{12}^2 \sum_{n=3,5,7,\cdots}^{\infty} \frac{r_n}{n^2(\mu r_n - \beta^2/k^2)} \approx a_{12}^2 \sum_{n=3,5,7,\cdots}^{\infty} \frac{1}{\mu n^2} = \frac{a_{12}^2}{\mu}\left(\frac{\pi^2}{8} - 1\right).$$

Substituting this result into (A4) yields the biquadratic equation

$$\left(\frac{\beta^2}{k^2}\right)^2 - \frac{\beta^2}{k^2}\left[\frac{\mu^2 - \kappa^2}{\mu} + \mu r_1 + \frac{a_{12}^2}{\mu}\right] + \mu r_1\left(\frac{\mu^2 - \kappa^2}{\mu}\right) = 0. \tag{A5}$$

which is the desired result.

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Experimental and Computed Four Scattering and Four Noise Parameters of GaAs FET's Up to 4 GHz

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***Abstract*—The four scattering parameters, operating in the pinch-off mode, of a Schottky-barrier-gate FET (MESFET) are investigated with the aid of an appropriate equivalent circuit. The dependence of the electron drift velocity on the electric field of the channel has been simplified to be piecewise linear by Turner and Wilson. Hot electron effects have therefore been neglected. The four noise parameters of the device have also been computed using the noise sources of van der Ziel. All computed parameters are compared with their measured values in the frequency region 0.5–4 GHz. Investigated GaAs FET's are commercial units.**

I. Introduction

An appropriate equivalent circuit of a GaAs FET valid up to 4 GHz is presented here. The channel of this transistor is n-doped, with a carrier concentration of about 3×10^{16} cm^{-3}. The gate length of the device is about 4 μm and the channel width is about 360 μm. The GaAs FET is mounted in a microstrip package with three terminals. The equivalent circuit of such an FET valid up to 4 GHz has been computed using a computer-aided optimization program, based on the classical gradient method. The established network equations have been analyzed using the SYMBAL computer language [8]. Theoretical work on noise in FET's has been described by van der Ziel [2], [3] and Leupp and Strutt [9]. These data have been applied to the intrinsic GaAs FET without any modification.

The computations on the small-signal behavior as well as on the noise behavior, i.e., the four scattering parameters and the four noise parameters, have been made neglecting hot electron effects [15],

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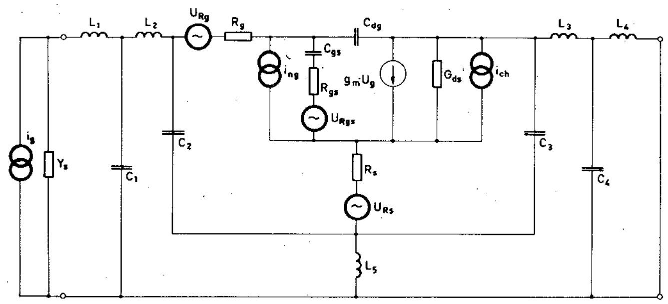

Fig. 1. Complete equivalent circuit of the GaAs FET valid up to 4 GHz. V_d = 4 V; I_d = 5 mA; R_{gs} = 1.95 Ω; C_{gs} = 0.4865 pF; C_{dg} = 0.265 pF; g_m = 10.85 mmho; G_{ds} = 2.124 mmho; R_g = 6.4560 Ω; R_s = 23.6 Ω; C_3 = 0.283 pF; L_1 = 0.439 nH; L_2 = 0.428 nH; L_3 = 0.2828 nH; L_4 = 0.4014 nH; L_5 = 1.41 nH; C_1 = 0.0506 pF; C_2 = 0.0564 pF; and C_4 = 0.158 pF.

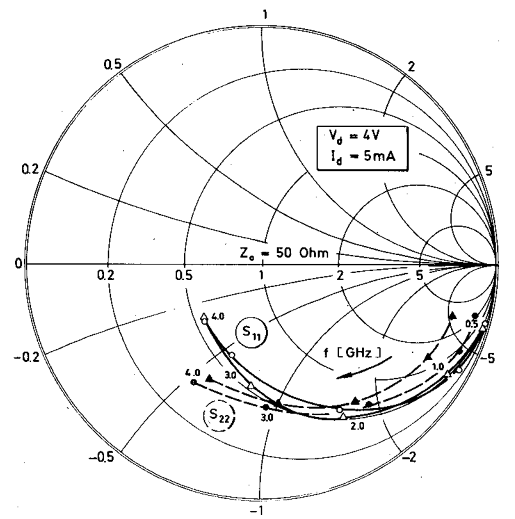

Fig. 2. S_{11} and S_{22} parameters of the GaAs FET as functions of the frequency. O: measured; Δ: computed.

[16], since in our sample the channel length L is such that, under the above conditions, the carrier mobility may be considered as a constant [13], [14].

II. Small-Signal and Noise Parameters of the GaAs FET's

The complete equivalent circuit of a GaAs FET is represented in Fig. 1. It consists of the intrinsic transistor, of the extrinsic elements R_g (gate metallization resistance), R_s (source resistance), and C_3 (extrinsic drain-to-source capacitance); and of the package elements [5] L_1, L_2, L_3, L_4, L_5 (inductances of the wires); and of the package capacitances C_1, C_2, and C_4.

The elements of this equivalent circuit have been computed using a computer-aided optimization program. In the optimization procedure the model circuit was not changed, only the values of its elements. The scattering parameters as measured are compared with their computed values using the equivalent circuit of Fig. 1.

Fig. 2 represents the computed S_{11} and S_{22} parameters in comparison with their measured values, as a function of the frequency. The computed as well as the measured S_{12} and S_{21} parameters are shown in Fig. 3. A good concordance is obtained between the measured values of the four scattering parameters and the computed values from the model circuit.

In order to describe the noise behavior of the GaAs FET's, a number of noise sources has to be taken into account in the equivalent circuit. In Fig. 1 these noise sources are shown. The two most important noise sources are the intrinsic noise sources i_{ch} (channel noise source) and i_{ng} (induced gate noise source), given by [2], [3]:

$$\langle i_{ch}^2 \rangle = 4KTg_m f_1 \Delta f \tag{1}$$

$$\langle i_{ng}^2 \rangle = \langle i_{no}^2 \rangle + \frac{4KT}{g_m} \cdot \omega^2 \cdot C_{gs}^2 f_2 \cdot \Delta f \tag{2}$$

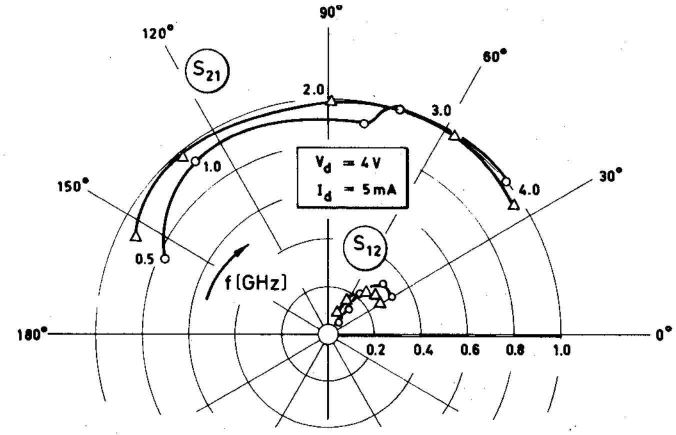

Fig. 3. S_{21} and S_{12} parameters of the GaAs FET as functions of the frequency. O: measured; Δ: computed.

where

K	Boltzmann constant;
T	absolute temperature;
f_1, f_2	constants depending on the biasing conditions of the transistor;
Δf	noise bandwidth;
$\langle\ \rangle$	mean value;
$\langle i_{no}^2 \rangle$	low-frequency value of $\langle i_{ng}^2 \rangle$;
g_m	the transconductance of the device;
ω	circular frequency.

These two noise sources have the same origin [3]. Therefore, a correlation factor C_{or} (purely imaginary) exists between them:

$$jC_{or} = \frac{\langle i_{ng}^* \cdot i_{ch} \rangle}{(\langle i_{ng}^2 \rangle \cdot \langle i_{ch}^2 \rangle)^{1/2}} \tag{3}$$

where * indicates the complex conjugate. The three resistances R_g, R_{gs}, and R_s generate the thermal noise voltage U_{Rg}, U_{Rgs}, and U_{Rs} shown in Fig. 1. A further thermal noise source i_g, caused by the source admittance $Y_s = G_s + jB_s$, also has to be considered:

$$\langle i_g^2 \rangle = 4kT \cdot G_s \cdot \Delta f. \tag{4}$$

The noise figure F of the GaAs FET can be expressed using the circuit of Fig. 1, by

$$F = 1 + \frac{\langle (i_{Rgo} + i_{Rgso} + i_{ngo} + i_{Rso} + i_{cho})^2 \rangle}{\langle i_{go}^2 \rangle}$$
$$= 1 + \frac{\langle i_{Rgo}^2 \rangle + \langle i_{Rgso}^2 \rangle + \langle i_{ngo}^2 \rangle + \langle i_{Rso}^2 \rangle + \langle i_{cho}^2 \rangle + 2 \operatorname{Re} \langle i_{ngo}^* \cdot i_{cho} \rangle}{\langle i_{go}^2 \rangle} \tag{5}$$

with i_{Rgso}, i_{Rgo}, i_{ngo}, i_{Rso}, i_{cho}, and i_{go} being the short-circuited output noise currents generated, respectively, by the noise sources u_{Rgs}, u_{Rg}, i_{ng}, u_{Rs}, i_{ch}, and i_g.

The noise figure F of (5) has been calculated using a computer-aided program for several values of the source admittance. Therefore, the four noise parameters $F_{\min}$ (minimum noise figure), $Y_{on} = G_{on} + jB_{on}$ (optimum source admittance with respect to noise) and R_n (noise resistance), described in (6) can be determined:

$$F = F_{\min} + \frac{R_n}{G_s} [(G_s - G_{on})^2 + (B_s - B_{on})^2]. \tag{6}$$

Fig. 4 shows the computed minimum noise figure $F_{\min}$ compared with its measured value. Fig. 5 shows the computed value of the optimum source admittance Y_{on} and its measured value, as a function of the frequency. Finally, Fig. 6 shows the noise resistance R_n.

III. Conclusion

The four scattering parameters of a GaAs FET at microwave frequencies are considered. An appropriate equivalent circuit of the transistor valid up to 4 GHz has also been presented. It has also

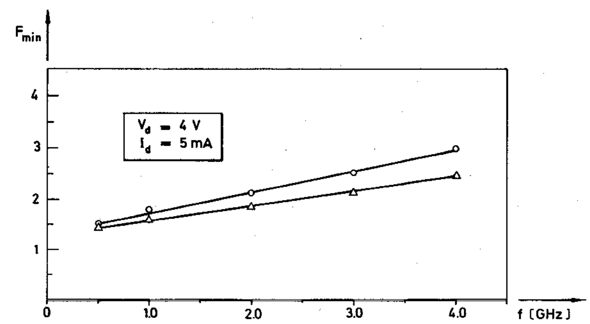

Fig. 4. Minimum noise figure $F_{\min}$ of the GaAs FET as a function of the frequency. ○: measured; △: computed.

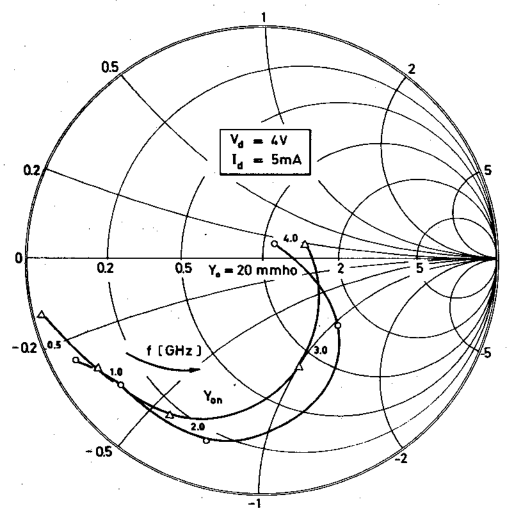

Fig. 5. Optimum source admittance Y_{on} of the GaAs FET with respect noise versus frequency. ○: measured; △: computed.

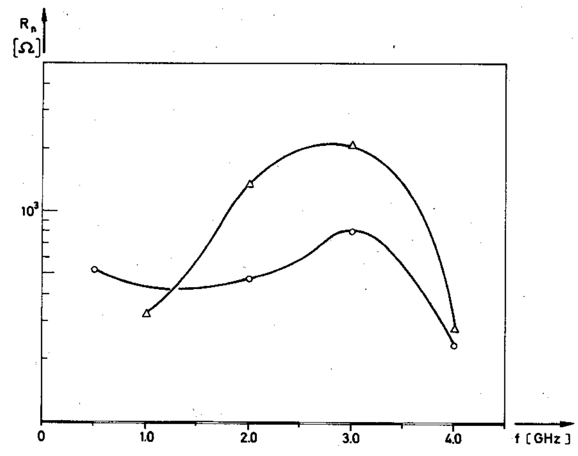

Fig. 6. Noise resistance R_n of the GaAs FET as a function of the frequency. ○: measured; △: computed.

been demonstrated that the compound noise parameters of the GaAs FET, based on the noise model of van der Ziel, are in good accordance with the measured results up to 4 GHz.

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An Automatic System for Simultaneous Measurement of Amplitude and Phase of Millimeter-Wave Fields

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Abstract—A measuring system for the simultaneous determination of amplitude and phase distributions of EM fields is described. The system employs the vibrating-dipole technique, which is based on the principle of modulated scattering. The phase-modulated wave scattered by the vibrating dipole contains information about the amplitude and phase of the EM field at the midpoint of the dipole's vibration.

The system has been made automatic by the inclusion of a servo loop. The measured free-space distributions compare well with those calculated from theory.

I. Glossary of Symbols Defined in [9]

E_0	Amplitude (real) of the reference wave in the waveguide.
E_s	Amplitude (real) of the back-scattered wave in the waveguide.
$\boldsymbol{F}(x,y,z)$	Dimensionless complex vector function of position.
F_x	Amplitude (real) of the x component of $\boldsymbol{F}$.
G	Constant of amplification and rectification.
J_n	Bessel function of the first kind and order n.
K	Constant of proportionality of the detector.
λ	Free-space wavelength.
ϕ_m	$=2\pi d/\lambda$, where d is amplitude of vibration of the dipole.
ϕ_p	Phase lag introduced by the phase shifter.
ϕ_x	Phase of the x component of $\boldsymbol{F}$.
ω_m	Angular frequency of vibration of the dipole.

II. Introduction

The knowledge of EM field distributions is essential in many problems, particularly in diffraction studies of apertures and obstacles. In some cases the amount of required experimental information is so vast that an automatic measuring system becomes indispensable.

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